Understanding the Opioid Syndemic in North Carolina: A Novel Approach to Modeling and Identifying Factors

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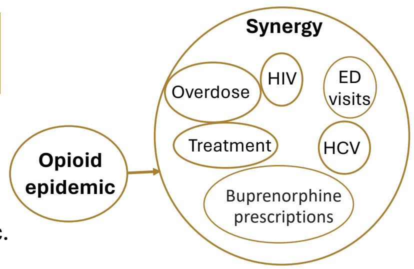
Motivation

- **Opioid epidemic** is a critical public health problem, e.g. in 2021 4041 NC residence died from opioid overdose

Challenge: no data to fully capture this complex problem. → **Need of an alternative approach**

We know that:

- Opioid epidemic is associated with HIV and HCV infections.
- Fatal and nonfatal overdose, ED admissions and treatment admissions are also linked to opioid misuse.
- These epidemics interact synergistically, forming a syndemic.



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Identify underlying factors that explain dependencies in the observed data.

Methodology

- **Bayesian Hierarchical Model:**
  - Observed count for outcome k in year j and in county i :

$$Y_{ijk} \sim Poisson(E_{ijk}\lambda_{ijk})$$
$$\log(\lambda_{ijk}) = \Gamma_k' F_{ij} + \epsilon_{ijk}$$
$$F_{ij} \sim ICAR - AR(1)$$
$$\epsilon_{ijk} \sim N(0, \sigma_k^2)$$

Factor Analysis

- allows us to design factors that yield **meaningful interactions** among predefined subsets of the the observed outcome.

- **Describe the opioid syndemic through 4 factors:**
  - Factor 1 to characterize **variation** shared by all **six outcomes: overall burden.**
  - Factor 2 to characterize **variation** shared by **Death (D) and ED visits (E)**, after accounting for F1: **overdose aspect** of the epidemic.
  - Factor 3 to characterize **variation** shared by **Treatment (T) and Buprenorphine (B)**, after accounting for F1: **treatment aspect** of the epidemic.
  - Factor 4 to characterize the remaining shared **variation** between **HCV (C) and HIV (I), after** accounting for F1: **infectious disease** elements of the epidemic.

Identifiability of $log(\lambda_{ij}) = \Gamma \mathbf{F}_{ij} + \epsilon_{ij}$

- A well-known challenge in factor analysis modeling is the identifiability[1] of model parameters, i.e., The likelihood $f(\boldsymbol{Y}_{ij}|\boldsymbol{\Gamma}, \mathbf{F}_{ij}, \boldsymbol{\epsilon}_{ij})$ is equivalent to $f(\boldsymbol{Y}_{ij}|\boldsymbol{\Gamma}\mathbf{H}, \mathbf{H}'\mathbf{F}_{ij}, \boldsymbol{\epsilon}_{ij})$ for any $m \times m$ orthogonal $\boldsymbol{H}$.
- **Common solution**: specify the loadings matrix to be lower triangular[2] ⇒ factors lacking meaningful interpretation.
- **Proposed solution**: specify the loadings matrix $\boldsymbol{\Gamma}$ as needed to quantify meaningful relationships among subsets of outcomes and employ LQ decomposition[3] on $\boldsymbol{\Gamma}$ to decompose it into a lower triangular matrix with **positive diagonal elements**, $\boldsymbol{L}$, and an orthogonal matrix, $\boldsymbol{Q}$, i.e., $\boldsymbol{log(\lambda_{ij})} = \boldsymbol{\Gamma F_{ij}} + \boldsymbol{\epsilon_{ij}} = \boldsymbol{LQF_{ij}} + \boldsymbol{\epsilon_{ij}} = \boldsymbol{L\,\widetilde{F}_{ij}} + \boldsymbol{\epsilon_{ij}}$, ensuring that the new model adheres to the identifiability constraints[4] of a factor model.

Illustration:

$\log(\lambda_{ijk}) = \Gamma_k' F_{ij} + \epsilon_{ijk}, k = 1, \ldots, 4$, written in matrix form:

Loadings matrix

$$\log(\boldsymbol{\lambda}_{ij}) \equiv \begin{bmatrix} \log(\lambda_{ijD}) \\ \log(\lambda_{ijE}) \\ \log(\lambda_{ijT}) \\ \log(\lambda_{ijB}) \\ \log(\lambda_{ijC}) \\ \log(\lambda_{ijI}) \end{bmatrix} = \begin{bmatrix} 1 & \gamma_{12} & 0 & 0 \\ \gamma_{21} & 1 & 0 & 0 \\ \gamma_{31} & 0 & 1 & 0 \\ \gamma_{41} & 0 & \gamma_{43} & 0 \\ \gamma_{51} & 0 & 0 & 1 \\ \gamma_{61} & 0 & 0 & \gamma_{64} \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} + \begin{bmatrix} \epsilon_D \\ \epsilon_E \\ \epsilon_T \\ \epsilon_B \\ \epsilon_C \\ \epsilon_I \end{bmatrix}$$

↓

Must be identifiable ≡ *lower triangular with positive diagonal elements*

↙ ↘

L Q

factorization

Algorithm: Metropolis-Hastings Update of Loadings

0\. Let $\boldsymbol{\Gamma}$ denote the current value of the loadings with $\boldsymbol{\Gamma} = \boldsymbol{LQ}$.

1\. Propose a new value γ_{km}^* using a normal random walk, centered at the current value γ_{km}. Let $\boldsymbol{\Gamma}^*$ be the loadings matrix containing the proposed γ_{km}^*.

2\. Perform the LQ-decomposition of $\boldsymbol{\Gamma}^*$:

Input($\boldsymbol{\Gamma}^*$)

- Let $\boldsymbol{\Gamma}^{*\prime} = [\Gamma_2^{*\prime} \,|\, \Gamma_2^{*\prime}]$.
- Apply the QR Gram-Schmidt process to get $\boldsymbol{Q}^*$ and $\boldsymbol{R}_1^*$ such that $\boldsymbol{\Gamma}_1^{*\prime} = \boldsymbol{Q}^* \boldsymbol{R}_1^*$.
  - Compute $\boldsymbol{R}_2^* = \left[\boldsymbol{Q}^{*\prime} \cdot \Gamma_2^{*\prime}\right]$.
  - Let $\mathbf{R}^* = [\boldsymbol{R}_1^* | \boldsymbol{R}_2^*]$.

Output ($\boldsymbol{L}^* = \boldsymbol{R}^{*\prime}, \boldsymbol{Q}^* = \boldsymbol{Q}^*$)

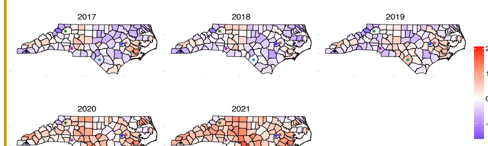
3\. Compute the Metropolis-Hastings acceptance probability:

$$\tau = \min\left(1, \frac{f(\boldsymbol{y}|\boldsymbol{L}(\boldsymbol{\Gamma}^*), \boldsymbol{Q}(\boldsymbol{\Gamma}^*), \boldsymbol{F}, \boldsymbol{\epsilon})\pi(\Gamma^*)}{f(\boldsymbol{y}|\boldsymbol{L}(\boldsymbol{\Gamma}), \boldsymbol{Q}(\boldsymbol{\Gamma}), \boldsymbol{F}, \boldsymbol{\epsilon})\pi(\Gamma)}\right).$$

4\. Set $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^*$ with probability τ .

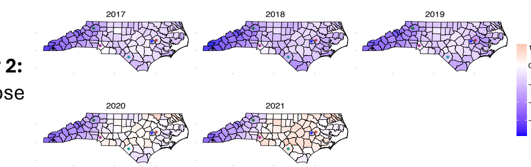
Results

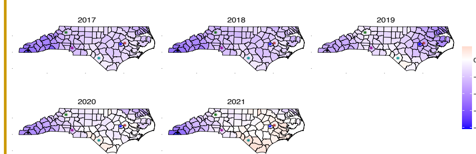
Posterior mean estimates of the four latent factors from 2017 to 2021. **Brown** star represents Clay county, **blue** star represents Green county, **dark green** star represents Mecklenburg county, **dark orange** star represents Pitt county, **dark magenta** star represents Robeson county **dark cyan** star represents Wilkes county.



Factor 1: Burden

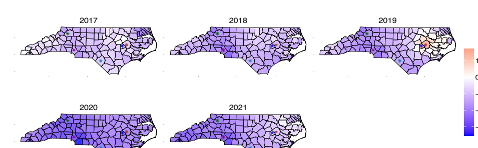
Factor 2: Overdose





Factor 3: Treatment

Factor 4: Infectious diseases



Next steps

- Combine county-level data with zip code-level data to understand the opioid epidemic at a more granular level.
- Use loadings that vary geographically.

References

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