Modeling of Wind Speed and Wind directions through a conditional approach

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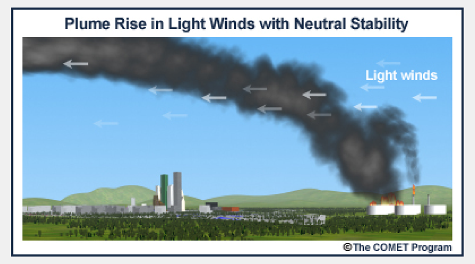
Motivation



Source: www.lawrencehallofscience.org

Source: www.wsp.com

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- **Scientific question:** How to accurately model wind speed and wind direction?
- **Applications**: Modelling of air pollution; Building engineering; Clean energy. for coastal nuclear power plants
- **Goal**: Construct a bivariate model for jointly modeling wind speed and wind direction.

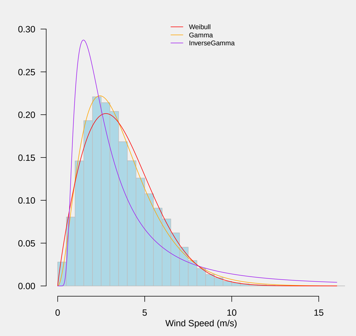
Modeling wind speed, wind direction and their dependence

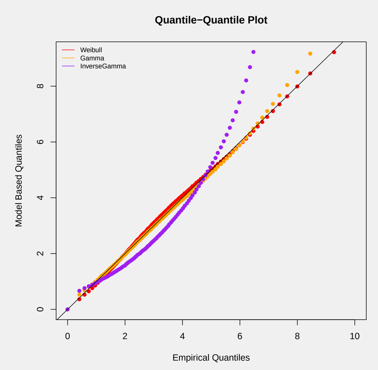
Wind speed probability distribution:

- Two parameter Weibull distribution:

$$f(v;\alpha,\beta) = \left(\frac{\alpha}{\beta}\right)\left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[\left(-\frac{v}{\beta}\right)^{\alpha}\right],$$

where α (shape) and β (scale) > 0



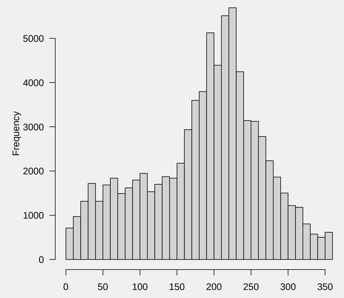


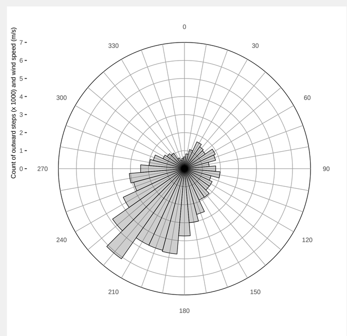
Wind direction probability distribution

- von Mises distribution

$$f(\phi;\mu,\kappa) = \frac{1}{2\pi I_0(\kappa)} \exp[\kappa\cos(\phi-\mu)],$$

where $\kappa \geq 0$ is the concentration parameter and $0 \leq \mu < 2\pi$ corresponds to the mean direction.

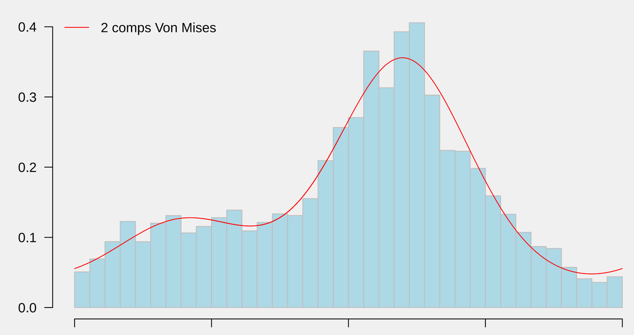




- Mixture of von Mises distributions

$$f_{mix}(\phi;\mu,\kappa,\omega) = \sum_{j=1}^{N} \omega_j f(\phi;\mu_j,\kappa_j),$$

where N is the number of components, ω_j are weights such that $\sum_{j=1}^{N}\omega_j = 1$



Proposed statistical model

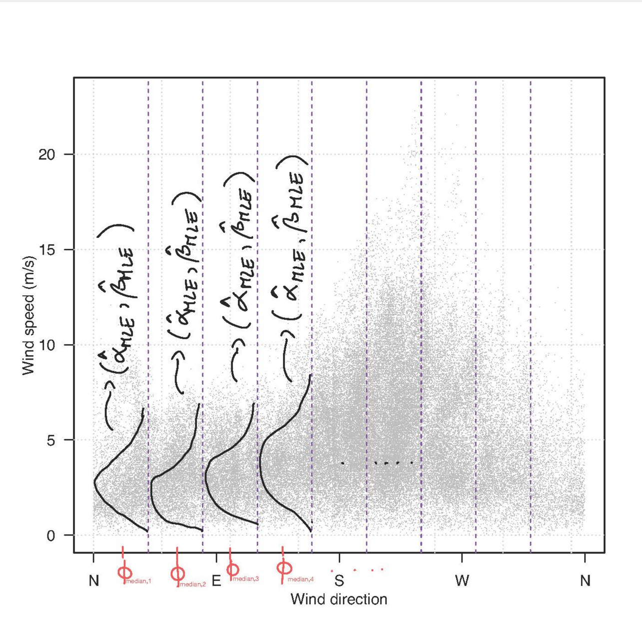
Decompose the joint distribution of wind speed, V, and wind direction, Φ, into the product of a conditional distribution and a marginal distribution, e.g.,

$$[V,\Phi] = \underbrace{[V|\Phi]}_{\substack{\text{directional}\\\text{wind speed}\\\text{distribution}}} \cdot \underbrace{[\Phi]}_{\substack{\text{mixture of}\\\text{von Mises}\\\text{distribution}}}$$

Estimating the directional wind speed distribution

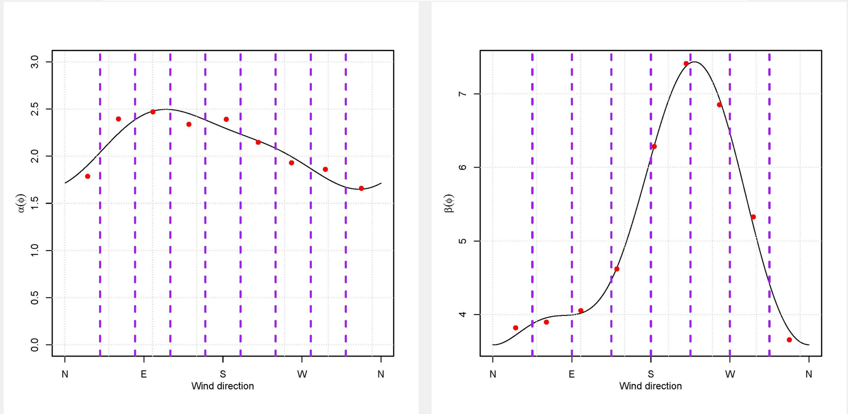
Stage 1: reduce the wind data to the estimates of the parameters of the Weibull distribution and a summary statistics of wind direction (e.g. medians):

- Divide the wind direction data into bins.
- In each bin, fit a Weibull distribution to the wind speed data using MLE;
- From each bin, extract the parameter estimates $\{\hat{\alpha}_i, \hat{\beta}_i\}_{i=1}^{N}$, their SEs, and summary statistics for the wind direction $\{\tilde{\phi}_i\}_{i=1}^{N}$.

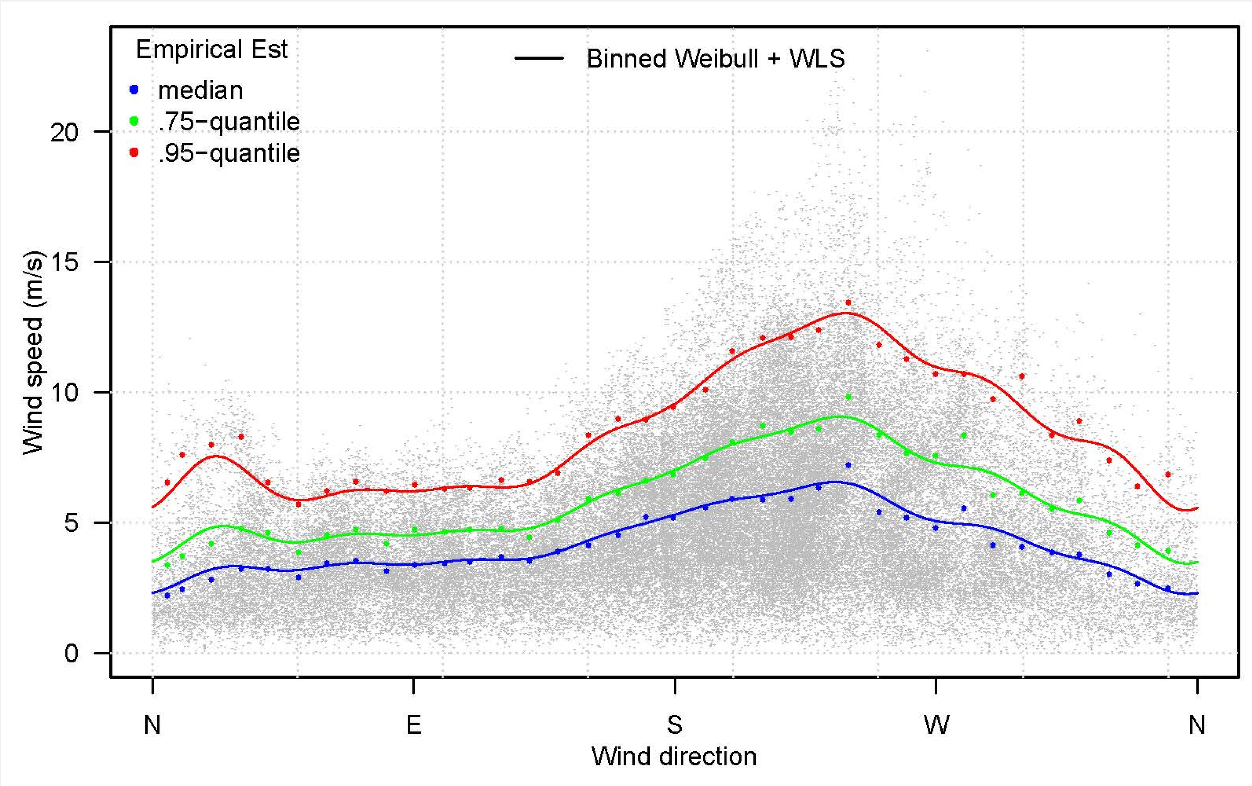


Stage 2: estimate the dependence structure, i.e. $\alpha(\phi)$ and $\beta(\phi)$, using harmonic regression via weighted least squares:

- Use $\{\tilde{\phi}_i, \hat{\alpha}_i\}_{i=1}^{N}$ as data points;
- Regress $\{\hat{\alpha}\}_{i=1}^{N}$ on $\{\tilde{\phi}_i\}_{i=1}^{N}$ using harmonic regression via weighted least squares (WLS).
- Regress $\{\hat{\beta}\}_{i=1}^{N}$ on $\{\tilde{\phi}_i\}_{i=1}^{N}$ using harmonic regression via WLS.



Represent the directional wind speed distribution through conditional quantile curves:

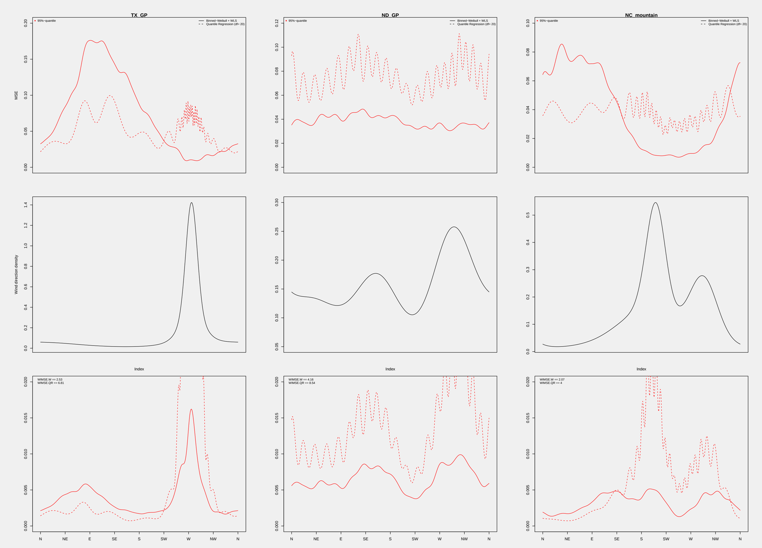


Simulation results

- **Aim**: Compare the performance of the Binned Weibull - WLS model to that of Quantile Regression;
- **Simulation setup**: Simulate 500 replicates, each replicate of size 7360 ($\sim$ 10 years of 3-hourly data) of $[V,\Phi]$ that mimic a wide range of wind speed and direction distributions;
- **Performance measures**:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$
$$\text{WISE} = \int\left(Y-\hat{Y}\right)^2 f_\Phi(\phi)\,d\phi$$

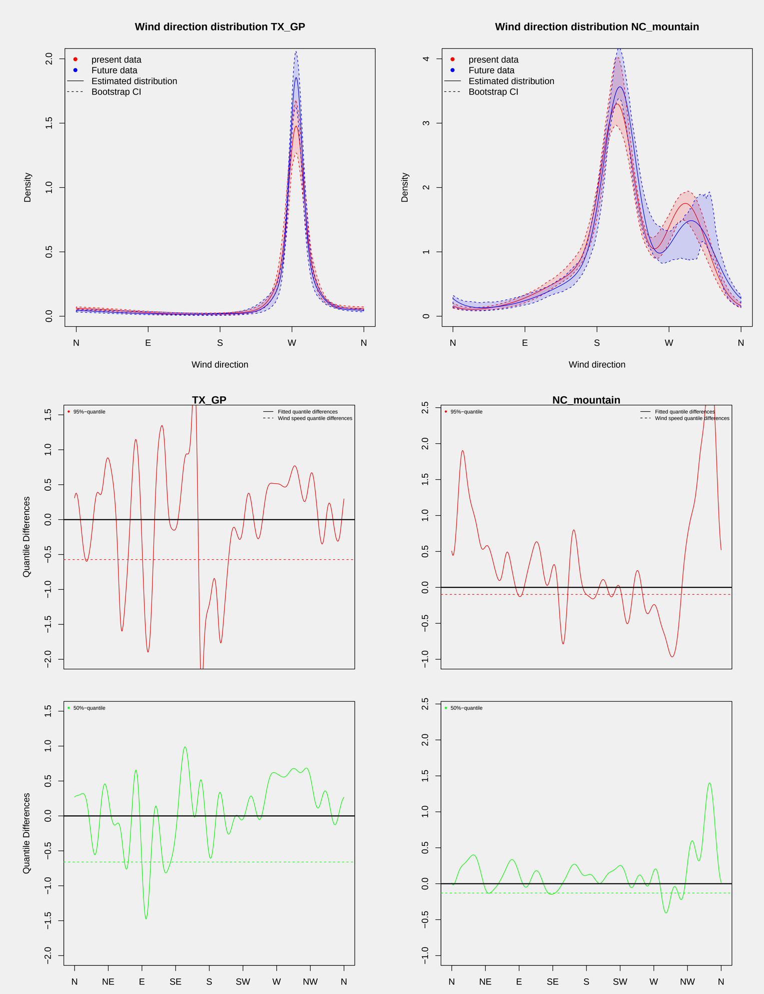
where Y and $\hat{Y}$ denote the true and estimated **conditional quantiles** at a given quantile level and an angle.



- For the most parts the Binned - Weibull WLS model outperforms Quantile Regression.

Estimating changes in wind speed and wind direction distributions

Use the Community Climate System Model (CCSM) outputs of 3-hourly wind speed and direction under a present (years 1995 - 2004) and future (years 2085-2094) climate scenarios.



- Changes in $[\Phi]$: Fit a three component von Mises mixture distribution to the present and future wind directions.
- Changes in $[V|\Phi]$: Fit the Binned-Weibul-WLS model to the present and future wind speed and wind direction and calculate the differences in the future and present quantiles (solid lines).
- Changes in $[V]$: Fit a Weibul distribution to the present and future wind speed data and calculate the differences in the future and present quantiles (dashed lines).
- The Binned-Weibul-WLS model allows us to detect differences in present and future wind speed and direction distribution.

Discussion and future work

- We propose a **conditional approach** to jointly model wind speed and wind direction.
- Application in climate change: our method allows us to detect differences in present and future wind speed and direction distribution.
- **Future work:** Spatio - Temporal Modeling of Wind Speed and Wind Direction by including
  - Seasonality
  - Spatial modeling